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— SECTION A —

PHYSICAL SCIENCES

Contents

THE USE OF CUMULATIVE RESISTANCE IN EARTH-RESISTIVITY SURVEYS[1]

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Abstract

When the soil is assumed to consist of two layers—the upper of resistivity ρ_1 and the lower of resistivity ρ_2—and cumulative resistances are calculated by adding or integrating the earth-resistivity functions for intervals that are a fraction of the thickness of the upper layer, a practically linear relation is obtained between the cumulative resistance and the electrode spacing until the distance between the electrodes is equal to the thickness of the upper material. Should one of the materials be at least twice as conducting as the other, the extent of the deviation from the linear law enables the determination of the depth of the upper stratum and of the ratio between the resistivities of the two layers. When three layers are present and the middle layer is at least twice as thick as the top stratum, the thicknesses may be deduced from the two departures of the cumulative resistances from the linear law. Since these conclusions are based on the theory of the individual apparent resistivity of stratified ground at various electrode spacings, they have the same range of application as the earth-resistivity curves, but the occurrence of straight line graphs facilitates the plotting and the interpretation of results based on a necessarily limited number of measurements.

Introduction

In recent years the Wenner method of electrical resistance surveying has been in frequent use for the detection of water in the ground and for the determination of the electrical conductivity of the subsoil in general. Four rods, each about 50 cm. long and 2 cm. in diameter, are driven into the ground along a straight line, at equal distances from one another. When the outer electrodes, made of solid metal and called current terminals, are connected to a battery or to a portable d-c. generator, a current of i amperes enters the ground at one electrode and leaves it at the other. The potential difference v, measured between the inner or potential terminals, contained in porous porcelain pots, is corrected, if necessary, for voltages that are the result of earth-currents, by reversing current and voltage in rapid succession.

On the assumption that the ground is of uniform composition and structure, down to a great depth, the specific resistivity of the soil is given by the formula

$$\rho = 2\pi a \frac{v}{i},$$

where a is the distance between neighbouring electrodes. Provided that the

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spacing a is expressed in cm., the formula for ρ gives the resistivity between electrodes of 1 sq. cm. area each, 1 cm. apart; when a is expressed in metres, the result obtained refers to the 100 times smaller resistance between opposite faces of a cube of a metres side (ohm-m.).   Determinations of the resistivity at any distance a give a constant result for ρ as long as the current flows through one and the same kind of soil.   But if a second approximately horizontal bed of material begins at a certain depth h_1 below the surface, more and more current will flow through it as the distance between the electrodes is increased from about h_1 to larger values.   A graph in which each measured resistance is plotted as abscissa from a vertical axis calibrated in distance, and later in depth, shows, instead of a vertical straight line, a gradual transition from the initial value ρ_1 to a second, smaller or greater, value ρ_2.   When the lower bed is a better conductor, direct current tends to flow at great depth rather than along the surface, and the resistance measured at large electrode spacings is equal to that of the better conductor.   In the presence of an insulator at great depth, only a relatively small portion of the current enters the lower bed, and while the resistance $\rho_s = 2\pi a v / i$, the so-called apparent resistivity, increases when the electrodes are moved farther apart, the value ρ_2 characteristic of the poor conductor is not reached.   In each alternative the change in resistance is most often gradual, and the depth h_1 at which the deeper layer occurs cannot be found at first sight from the graph representing the resistance as a function of the distance separating the electrodes.   Should the conductivity of one of the two media be markedly different, the resistance graph for a soil consisting only of two beds will exhibit a point of inflection, and the corresponding distance a on the horizontal axis is taken as the depth at which the second mineral replaces the surface bed (5).   Where a good conductor is underlain by more than one material with much higher but not infinite resistance, a U-shaped curve is often obtained, and the electrode spacing for the lowest value of the resistance reached is sometimes taken as the approximate thickness of the surface layer (5).

Frequently, however, the resistance within the same bed of rocks fluctuates, depending upon the depth, and the resistance graph presents peaks that do not correspond to a complete change in rock formation.   At some locations, on the other hand, a curve showing the resistivity at various depths may be smoothly rounded, with no pronounced inflection or trough, despite the presence of two or more different layers.

It has been reported that more reliable results are secured by means of an empirical procedure, in which the resistances obtained at each one of the smaller spacings a, $2a$, etc., are added to the value at the next greater distance and the sums are plotted as a function of the distance a between the electrodes (5).   In view of the widening field of application of resistivity surveys it was thought useful to ascertain how far the new method can be justified by theoretical considerations (5, 7).

The Cumulative Resistance Curve

(a) Two Different Layers are Present

When only two parallel beds have to be taken into account, the upper one having the resistivity ρ_1 and the thickness h_1, the lower, the resistivity ρ_2 and unlimited depth, the total resistance presented to the flow of current is given by the formula

$$\frac{v}{i} = \frac{\rho_1}{2\pi a}\left\{1 + 4\sum_{n=1}^{\infty} k_2^n \left[\frac{1}{\sqrt{1 + \left(2n\frac{h_1}{a}\right)^2}} - \frac{1}{2\sqrt{1 + \left(n\frac{h_1}{a}\right)^2}}\right]\right\},$$

where

$$k_2 = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} = \frac{\dfrac{\rho_2}{\rho_1} - 1}{\dfrac{\rho_2}{\rho_1} + 1}.$$

As a consequence of the presence of the fraction h_1/a in the sums representing the resistance, the quotient v/i is no longer independent of the distance a unless this distance is extremely large (3). For carrying out measurements, the distances a must, of course, be greater than the depth within which the roots of plants exert a strong influence on the properties of the soil. The theory assumes, moreover, that the length of the electrodes is small in comparison with their spacing. When $2\pi av/i$, with a value between ρ_1 and ρ_2, is called the apparent resistance, ρ_s, and $u = a/h_1$, the formula becomes

$$\frac{\rho_s}{\rho_1} = 1 + 4\sum_{1}^{\infty} k_2^n \left[\frac{u}{\sqrt{u^2 + 4n^2}} - \frac{u}{2\sqrt{u^2 + n^2}}\right].$$

The curve representing the measured values as a function of a will agree with one of the curves computed for the many possible values of k_2, on the assumption that the value of ρ_1, the resistivity of the upper layer, is taken equal to unity (Fig. 1). When the curve representing the measured values ρ_s as a function of the distance a rises rapidly as a increases, the bottom layer is a poor conductor, and the value of k_2 must be large (Table I). Should the material below the top layer conduct electricity better than the bed into which the electrodes are placed, then the rate of descent of the curve gives an estimate of the conductivity; as a result of its great depth it produces a marked reduction in the resistance between the terminals. After the variable ρ_s has been plotted as a function of the electrode separation a in graphs for which different scales are used for ρ_s, and the scale for a has been varied, if necessary, in each of these diagrams, one of the curves will be found to agree exactly with one of the theoretical curves showing ρ_s/ρ_1 against a/h_1 for various parameters k_2, and to yield thereby the correct value of h_1 and ρ_1. However, different interpretations of each curve are theoretically justifiable, and the correct choice can be made only if the geological evidence is in agreement with the assumption that only two layers are present.

FIG. 1.   *Curves for Wenner method, representing composite or apparent resistivities $2\pi a v/i$ when the surface layer has a thickness h_1 and its resistivity ρ_1 is taken as unity, and cumulative resistances (unbroken line), both as a function of the electrode spacing $u = a/h_1$.*

TABLE I

CORRESPONDING VALUES OF k_2 AND k_3 FOR TWO AND FOR THREE LAYERS

$$k_2 = \frac{\dfrac{\rho_2}{\rho_1} - 1}{\dfrac{\rho_2}{\rho_1} + 1} \qquad\qquad k_3 = \frac{\dfrac{\rho_3}{\rho_1} - \dfrac{\rho_2}{\rho_1}}{\dfrac{\rho_3}{\rho_1} + \dfrac{\rho_2}{\rho_1}}$$

$\dfrac{\rho_2}{\rho_1} =$		0	1/10	1/7	1/3	3	7	10	∞
$k_2 =$		−1	−9/11	−3/4	−1/2	+1/2	+3/4	+9/11	+1
k_3 for $\dfrac{\rho_3}{\rho_2} =$	0	1	−1	−1	−1	−1	−1	−1	−1
	1/10	1	0	−3/7	−7/13	−29/31	−69/71	−69/71	−1
	1/3	1	7/13	4/10	0	−4/5	−20/23	−29/31	−1
	1	1	9/11	3/4	1/2	−1/2	−3/4	−9/11	−1
	3	1	29/31	10/11	4/5	0	−2/5	−7/13	−1
	10	1	99/101	69/71	29/31	7/13	3/17	0	−1
	∞	1	1	1	1	1	1	1	0

(Should the lowest of three layers be a very poor conductor in comparison with the material at the top, the value of k_3 is equal to unity when the second layer is a good conductor.   When the lowest layer is a good conductor but the intermediate layer a poor conductor with respect to the material at the top, k_3 equals −1.)

According to the new method, the value plotted in the graph at the distance $2\,a/h_1$ is made equal to the sum of the resistances ρ_s/ρ_1 measured at a/h_1 and at $2\,a/h_1$, the value at $3\,a/h_1$ is made equal to the sum of the three resistances measured up to that point, and so on. When plotted at equal intervals the curve of the cumulative resistances is approximately proportional to the area between the ordinary resistance curve (Fig. 1), the a/h_1 axis, and the ρ_s/ρ_1-axis. The accurate value of the area is obtained by making the steps $u = a/h_1$ very small so that an integration is permissible. The integral of the function ρ_s/ρ_1 is equal to

$$S = \int \frac{\rho_s(u)}{\rho_1}\,du = u + 4 \sum_{n=1}^{\infty} k_2^n \left(\sqrt{u^2 + 4n^2} - \tfrac{1}{2}\sqrt{u^2 + n^2}\right),$$

and since for $u^2/h_1^2 < 1$ and $n > 1$

$$\sqrt{1 + \frac{u^2}{b^2}} = 1 + \frac{1}{2}\frac{u^2}{b^2} - \frac{1}{8}\frac{u^4}{b^4} + \frac{1}{16}\frac{u^6}{b^6} - \frac{5}{128}\frac{u^8}{b^8} + \cdots$$

the area between two values u_α and u_β, both smaller than unity, becomes

$$S_\beta - S_\alpha = (u_\beta - u_\alpha) + \sum_{n=1}^{\infty} k_2^n \left(\frac{3}{16}\frac{u_\alpha^4 - u_\beta^4}{n^3} - \frac{15}{128}\frac{u_\alpha^6 - u_\beta^6}{n^5}\cdots\right).$$

As long as the variables u are smaller than unity, the sum on the right-hand side of the equation is negligible; the difference $S_\beta - S_\alpha$, that is, the increase in area, is the same for the same increase in $u = a/h_1$, and simply equal to $(u_\beta - u_\alpha)$, whatever the ratio between the resistances. When the areas are plotted as a function of a/h_1, with $u = 0$ as the starting point, a straight line passing through the origin is obtained. Near $u = a/h_1 = 1$, however, the terms in u^4 and u^6 become appreciable, at least those for which $n = 1$, unless, of course, k_2 is very small and the two rock beds conduct electricity almost equally well. The correction is more important the more the two materials differ in resistance, in particular the greater the resistance of the lower bed (k_2 positive). At the first point where the straight line law is visibly no longer obeyed, the separation between the electrodes is equal to the thickness at which the upper bed ends.

In practice, the measurements yield merely the values of a and ρ_s, proportional respectively to u and ρ_s/ρ_1, but the conclusions concerning the slope of the cumulative resistance curve remain valid provided that the spacing between the electrodes is increased by a fraction of h_1 at a time. The difference $\rho_1(S_\beta - S_\alpha)$ given by the measurements in the place of $S_\beta - S_\alpha$ enables the determination of ρ_1, once the depth h_1 and the value of $(u_\beta - u_\alpha)$ have become known. Since the scale chosen for the graphs will tend to be smaller when ρ_1 is large, the deflection from the straight line remains about the same whatever the resistance ρ_1 of the top bed.

At $u = a/h_1 = 1$, the cumulative resistance line has reached, instead of the value 1, the slightly smaller value 0.948 for $k_2 = -9/11$, 0.967 for $k_2 = -1/2$, and the slightly greater value 1.04 for $k_2 = +1/2$, and 1.07 for $k_2 = +9/11$.

An accuracy of 5% under field conditions is just sufficient to detect the deviation in the slope.

In practice, instead of using integrals, the apparent resistances are measured and added merely at finite intervals—equal, for instance, to one-third or one-fourth of the depth of the upper stratum. However, a graph of these cumulative resistances, constructed from the function representing the individual apparent resistances, ρ_s, shows that the conclusions drawn remain valid for the discontinuous summations, the change in slope being even more pronounced than for the integrated values (Fig. 1). Beyond the transition to the resistivity of the lower bed, the slope of the cumulative resistance becomes once more constant, at a distance equal to three or four times the thickness of the upper layer.

Another difference between the calculated and the measured values is that the theory assumes an abrupt change in conductivity between the upper stratum and its support. In nature, the change often is gradual, at any rate when it is caused or influenced by differences in moisture content. The consequence is that when, for instance, the lower material possesses a higher conductivity, the decrease in the apparent resistance with increased spacing exceeds the computed decrease, at least in the neighbourhood of $a/h_1 = 1$, and the more abrupt transition is followed by a stage of constant resistance. When a cumulative resistance curve is constructed on this basis, it must show sharper transitions, followed by more nearly linear variations than might be expected from the assumption of a sharp boundary surface.

(b) Three Different Approximately Horizontal Layers are Present

When at a certain depth below the surface the second bed of rocks ends and a new material takes its place and extends down to very great depths, there is a second transition in the measured individual resistances and a tendency to approach the value characteristic of the deepest layer, at least if the second layer is more than half as thick as the upper stratum.

When the second layer has a resistivity that is several times as great as that of the top layer, the curve representing the apparent resistance as a function of the distance between the electrodes has a maximum that is reached at greater distances the greater the resistivity of the underlying third bed; when the second layer is a better conductor than the material at the top, the curve possesses a minimum at a finite distance, unless the third layer is a still better conductor than the first. The peak is sharp when the middle bed, without being an insulator, has a very much higher resistivity, and the lowest bed a very much smaller resistivity, than the uppermost layer; the troughs are distinct when the middle stratum, without being a perfect conductor, is a better, and the lowest stratum, without being an insulator, is a poorer, conductor than the uppermost material. In the extreme alternatives, that is, with the middle layer an insulator, or the middle layer a perfect conductor, no peaks or troughs are observed. As is to be expected, the trough

obtained when the middle layer is three times as conducting as the top layer is much narrower than the peak obtained when the middle layer is three times as resistant as the top layer. But the point of inflection caused by the presence of the second material, as well as the positions of the peaks and troughs, is in general difficult to locate with an accuracy sufficient to indicate the surface of separation, and it is advisable to determine what help is given by the integrated values.

When it is necessary to consider three layers, the composite resistivity follows from the formula

$$\frac{\rho_s}{\rho_1} = 1 + 4 \sum_{n=1}^{\infty} k_2^n \left\{ \frac{1}{\sqrt{1 + 4n^2h_1^2/a^2}} - \frac{1}{2\sqrt{1 + n^2h_1^2/a^2}} \right\}$$

$$+ 4(1 - k_2^2)k_3 \sum_0^{\infty} (n + 1)k_2^n \left\{ \frac{1}{\sqrt{1 + 4(H_1 + nh_1)^2/a^2}} - \frac{1}{2\sqrt{1 + (H_1 + nh_1)^2/a^2}} \right\}$$

$$+ 4(1 - k_2^2)k_3^2 \sum_0^{\infty} f(k, n) \left\{ \frac{1}{\sqrt{1 + 4(H_2 + nh_1)^2/a^2}} - \frac{1}{2\sqrt{1 + (H_2 + nh_1)^2/a^2}} \right\},$$

where

$$k_3 = \frac{\dfrac{\rho_3}{\rho_1} - \dfrac{\rho_2}{\rho_1}}{\dfrac{\rho_3}{\rho_1} + \dfrac{\rho_2}{\rho_1}}$$

$$f(k,n) = \frac{n}{2}(n + 1)(1 - k_2^2)k_2^{n-1} - (n + 1)k_2^{n+1}$$

$$H_1 = h_2 + h_1$$

$$H_2 = 2h_2 + h_1.$$

Similar additional expressions, which involve $(3h_2 + h_1)/a$, $(4h_2 + h_1)/a$ etc., complete the equation but tend toward zero as long as the distances are moderate. The area between the ρ- and a-axis now is the sum of several terms (Ref. 3, and related papers 4, 11, 12). The first terms are identical with the ones found for two layers and give a linear increase in area until $a = h_1$. The second term may be written after the introduction of $u_1 = \dfrac{a}{h_2 + h_1} = a/H_1$

$$4(1 - k_2^2)k_3 \sum_0^{\infty} (n + 1)k_2^n$$

$$\times \left\{ \sqrt{u_1^2 + 4(1 + nh_1/H_1)^2} - \tfrac{1}{2}\sqrt{u_1^2 + (1 + nh_1/H_1)^2} \right\}$$

$$= 4(1 - k_2^2)k_3 \sum_0^{\infty} (n + 1)k_2^n \left(1 + \frac{nh_1}{H_1}\right)$$

$$\times \left\{ 2\sqrt{1 + \tfrac{1}{4}\left(\frac{u_1}{1 + nh_1/H_1}\right)^2} - \tfrac{1}{2}\sqrt{1 + \left(\frac{u_1}{1 + nh_1/H_1}\right)^2} \right\}$$

The sums converge even more poorly than those used for computing the apparent resistance of soils consisting of three different beds; but advantage may be taken of the conditions that the area is necessarily zero at the origin, and that the subsequent values are readily obtained by computing the dif-

ferences between the areas for equal differences $(u_\beta - u_\alpha)$, for instance, equal to 1/4.

The increment in area due to the presence of u_1 in the formula from the value $u_{1\alpha}$ to $u_{1\beta}$, both smaller than 1, is proportional to

$$\Delta = 4\sum_0^\infty (1 + n)k_2^n\left(\frac{3}{64}\frac{u_{1\alpha}^4 - u_{1\beta}^4}{(1 + nh_1/H_1)^3} - \frac{15}{512}\frac{u_{1\alpha}^6 - u_{1\beta}^6}{(1 + nh_1/H_1)^5} + \cdots\right),$$

and is, therefore, most often negligible. The same statement applies to the similar expressions containing $a/(2h_2 + h_1)$ and $a/(3h_2 + h_1)$, because when h_2 is several times larger than h_1, the value of

$$u_n = \frac{a}{nh_1 + h_2} = \frac{u}{n + h_2/h_1}$$

is small, and increases much more slowly than the corresponding values for the ratio $u = a/h_1$. The contribution depending on u_1 does not, therefore, necessarily prevent the detection of the difference from the linear law, which becomes appreciable, for two layers, at the point $u = 1$ when the areas are plotted as a function of u. On the other hand, since the term $1 + nh_1/(h_2 + h_1)$ in the denominator is smaller than n, more terms have to be taken into account for the correction than the number found sufficient for only two materials. The importance of the correction increases as u_1 approaches unity at the same time as u approaches the several times larger value $(1 + h_2/h_1)u_1$. Should the three beds be uniform and horizontal, then it is to be expected that the influence of u_1 causes another deviation from the straight line close
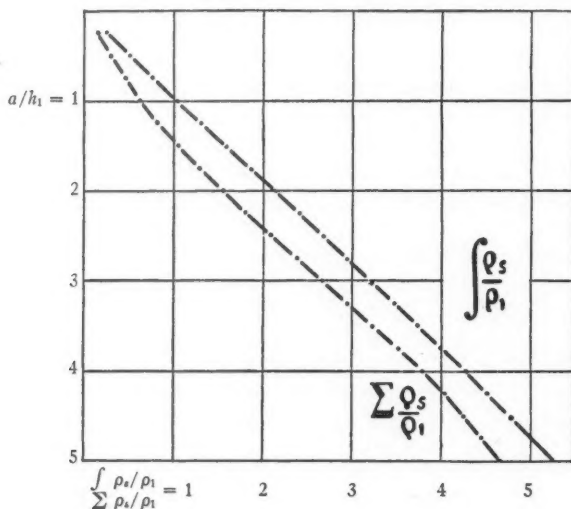

FIG. 2. Cumulative resistances for three layers as a function of the electrode spacing a/h_1 when $(h_1 + h_2)/h_1 = 4$.

to the point $u_1 = 1$, and reveals thereby the thickness of the second layer (Fig. 2), unless k_3 is very small, that is, unless the middle and the lowest stratum possess nearly the same conductivity.

Earth-resistivity Surveys and Grounding

In electrical engineering it is necessary to provide for adequate grounding of electrical circuits. Power plant equipment and transmission line towers are grounded, sometimes by running long rods and ribbons from the building or from the foot of the towers through the surrounding earth. A low footing resistance prevents high voltages from building up in the tower when these large structures are traversed by lightning, and reduces the risk of discharges between the tower and the line, and damage to the insulators. The determination of the resistance of a large cube of ground placed between metal electrodes gives a sufficiently accurate value of the resistance only when the ground is uniform to great depth, and even under these favourable conditions changes in moisture content may produce considerable alterations. By paying attention not only to the material in which the footing is established but also to the underlying beds and to the fluctuations of the water table, it should sometimes be possible to obtain low resistance to ground even under adverse geological and climatic conditions. Resistivity surveys are also of considerable assistance to the electrical industry because they enable the determination of the resistivity of the undisturbed soil.

The resistivity of any grounding bed depends in a large measure on the amount of moisture enclosed (10). For instance, when tested in dry air, a block of white marble, free from foreign material or visible differences in structure, was found to possess a specific resistivity of 2.6×10^{11} ohm-m. Under the influence of a confined volume of moist air, the resistance dropped within one day to 1/100 of its former value, then it decreased more slowly in the course of a week to 1.5×10^8 ohm-m. and remained at this value for at least two weeks, or until the air was replaced by a fresh sample containing moisture and carbon dioxide; then the resistance decreased to 4.2×10^7 ohm-m. Apparently the moisture penetrated rapidly into the pores, and in the presence of carbon dioxide the readily soluble bicarbonate of calcium was formed and the conductivity was improved. After the block had been soaked in water, the resistance decreased to 10^4 ohm-m. In dry air a gradual increase in the specific resistivity occurred until a value of at least 1.1×10^{13} ohm-m. was reached. The marble used in these tests had about 1000 pores per sq. cm., each of 15×10^{-8} sq. cm. cross-section. It took up about 1/1000 of its volume of water.

Similar wide variations are obtained with sandstone. Dry sandstone has a specific resistivity of 5×10^9 to 1×10^{11} ohm-m.; sandstone taken from the ground, 5×10^5 to 1.5×10^6 ohm-m.; and a sample previously soaked in water, 2×10^4 ohm-m. Dry coal is practically an insulator; moisture increases its conductivity.

The common rock-forming minerals, such as quartz, mica, and feldspar, are insulators but the rocks that contain them are poor conductors only if they are solid throughout. Most rocks and stones are more or less porous. The pores are filled in part with dilute solutions that are conducting. The conductivity of the rocks depends on the following conditions: (a) the pore volume, that is, the percentage of voids traversing the rock, usually referred to the total volume (solid plus pores); (b) the fraction of the pore volume occupied by water; (c) the conductivity and temperature of the water as it enters the pores or after contact with the rock; (d) the more or less orderly arrangement of the pores.

(a)   The porosity of the rocks depends on their age and the depth at which they lie. Porosity is extreme (70% to 90% of the total volume) for soft materials, such as peat and mud, very high (30 to 60%) for silt and clay, high (20 to 40%) for sand and gravel, moderate (5 to 20%) for ordinary sandstone, low for unfissured limestone, and very low (less than 1%) for dense crystalline limestones (marble) and igneous rocks.

(b) The extent to which the pores are filled with water depends on the size of the pores, and, in the first few feet below the surface, upon the annual amount of precipitation. It is difficult for the water to penetrate into very fine pores and to displace the air completely, even when the material is being flooded. In clay, silt, and fine sand, only a portion of the pore volume contributes to conduction, the remainder is filled with air and may be considered as forming part of the insulation. When the pores are coarser, as in ordinary sand—for which, by definition, the diameters of the grains exceed 0.2 mm.—or in gravel, the air is completely displaced by the water, so that the volume occupied by water may increase from 2 to 10% above the ground water level to 20 or 30% below the ground water level.

(c) Very elaborately purified water is reported to have a resistivity of about 200,000 ohm-m. at about 18° C.; ordinary distilled water, 1,200 ohm-m.; rain water collected in glass vessels, 800; surface water taken from brooks, streams, or lakes, 25 to 125 ohm-m.; and water obtained by pressure from cultivated alluvial soils (clay, gravel, sand), 5 to 15 ohm-m. Around room-temperature the resistance increases 2 to 3% for each decrease of 1° C. in temperature (5).

Rain or snow water that trickles through the ground contains small quantities of ammonia, nitric acid, and especially carbon dioxide, and is therefore able to dissolve some of the substances contained in the soil and in the rocks. If it has traversed granite and gneisses, the chief constituents found in solution will be sodium and potassium carbonates and bicarbonates. If sediments have been traversed, calcium and magnesium salts will dissolve, sometimes in such quantity that the pore volume is increased and a porous horizon formed near the upper boundary. Much common salt in water indicates that the precipitation has been in contact with marine deposits containing beds of salt. The amount of material dissolved varies with the size of the grains building up the sediment; finely grained materials, such as clay and silt,

have lower resistivity than coarse-grained loose materials, for the same water content, indicating that water is more salty in contact with fine grains. Water-bearing gravel has a high resistivity as a rule, a few hundred ohm-m.; clay, which is composed of flakes about 0.04 μ in thickness and 0.3 μ in diameter, may have a resistivity of only a few ohms.

(d) The effect of the shape and size of the pores in loose or compact rocks on the electrical resistance is difficult to evaluate. By assuming a very unfavourable and a very favourable method of connection between the pores, an upper and a lower limit may be obtained, in general, for the electrical resistance of the rock. It may be imagined, for instance, that each small cube of rock is surrounded by a film of water and that the cubes are placed in regular rows side by side. If each of the straight rows extending along the direction of the current is thought to be separated from the neighbouring rows by a very thin but perfect insulator, the resistance thus obtained must be greater than it was before the introduction of the insulating envelope.

The insertion, on the other hand, of thin but perfectly conducting surfaces between neighbouring layers of cubes and perpendicular to the direction of the current will result in a resistance that is lower than the real value. When the shortest distance separating a face of the cube from that of its neighbour is equal to γ, the specific resistivity of the solid equal to ρ_1, and that of the solution ρ_2, then the specific resistivity of the rock described has as its lower limit (3)

$$\rho_n = \frac{1}{1 + \gamma} \frac{\rho_1((1 + \gamma)^3 - \gamma) + \rho_2\gamma}{\rho_1\gamma(\gamma + 2) + \rho_2} \rho_2$$

and as its upper limi

$$\rho_s = (1 + \gamma)^2 \frac{\rho_1 + \gamma\rho_2}{\rho_1\gamma(\gamma + 2) + \rho_2((1 + \gamma)^3 - \gamma(\gamma + 2))} \rho_2 .$$

When γ is small and ρ_1, the resistivity of the minerals, very large, both expressions give the same limit, namely

$$\rho_0 = \frac{1 + 2\gamma}{2\gamma} \rho_2 .$$

Introduction of v, the fraction of the total volume occupied by pores, namely

$$v = \frac{(1 + \gamma)^3 - 1}{(1 + \gamma)^3}$$

when the side of the cube is taken as unity and γ is a small fraction, gives

$$\frac{\rho_0}{\rho_2} = \frac{3}{2v} - \frac{1}{2}.$$

This expression is valid until γ is as great as $3/10$, and v approaches 55%. When v is very small the resistance is inversely proportional to v, and the conductivity obeys the simple mixture rule. Measurements by Whitney and Means on quartz sand added to salt solution (14), by Haines on subsoil clay and clay soils (2), and by Muenger on sand and gravel wetted by salty water (6),

are in good agreement with this formula, except when less than about 6% water is present by volume and its distribution is far from uniform, since it will be retained in the wedges around lines of contact between the grains
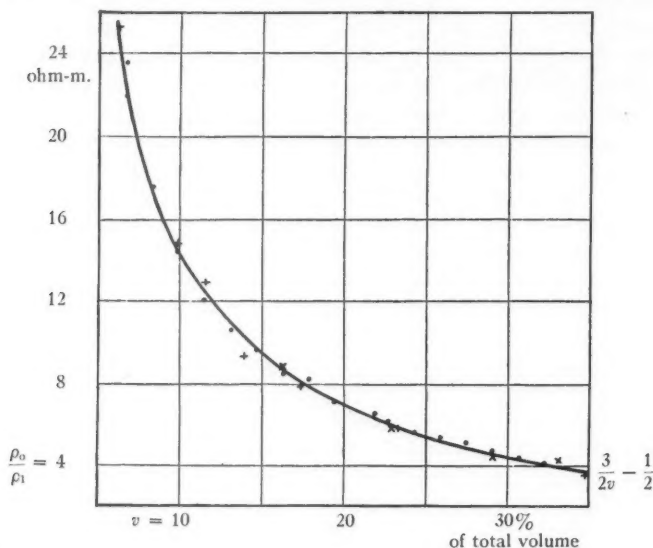

FIG. 3. *Resistivity of mixtures of sand* (+), *sand and gravel* (··), *or clay* (×), *with dilute solutions of salts in water (2, 6, 14).*

(Fig. 3). Some of the more recent measurements, repeated fortnightly for two years, give the following ranges for the resistivities of loose or solid beds (6):

	Ohm-m.
Stony ground, upper moraine	55 to 130
Stony ground, ground moraine	100 to 285
Gravel	
1/2 to 2 m. above water table	160 to 480
Part of time in ground water	110 to 560
50 m. above water table	420 to 955
Mud from lake bottom	15 to 25
Peat	22 to 35
Sandstone	60 to 120
Conglomerate, cemented	130 to 270
Limestone (Jurassic)	445 to 970

Detection of Water-bearing Horizons

The difference in the pore volume of unconsolidated deposits, such as clay, sand, and gravel, on the one side, and rocks, such as granite and unfissured limestone, on the other, is always such that even when the pores are completely filled with water the rocks are relatively bad conductors (1). The conductivity of wet sand and wet clay is many times as great as that of the same materials when dry, so that a bed of the same material, dry at the top
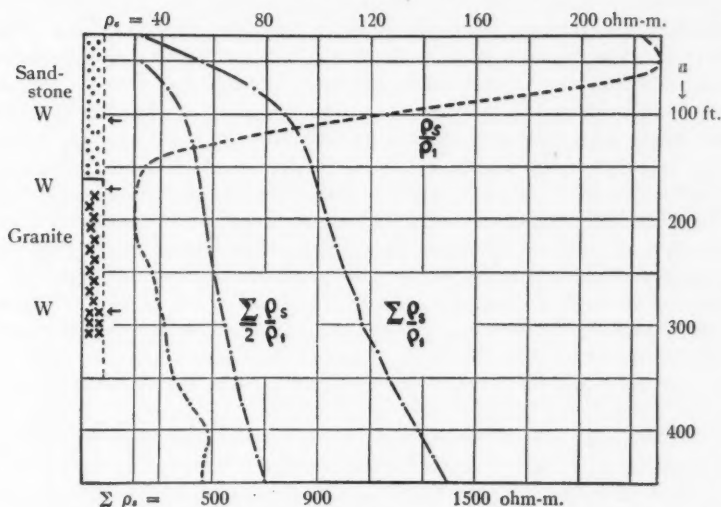
FIG. 4.   *Earth-resistivity curve and cumulative resistances from a survey in Matabeleland, west of Bulawayo (Southern Rhodesia), in sandstone and granite (8). W = water level. In the graphs the resistivity ρ_1 is taken as unity.*
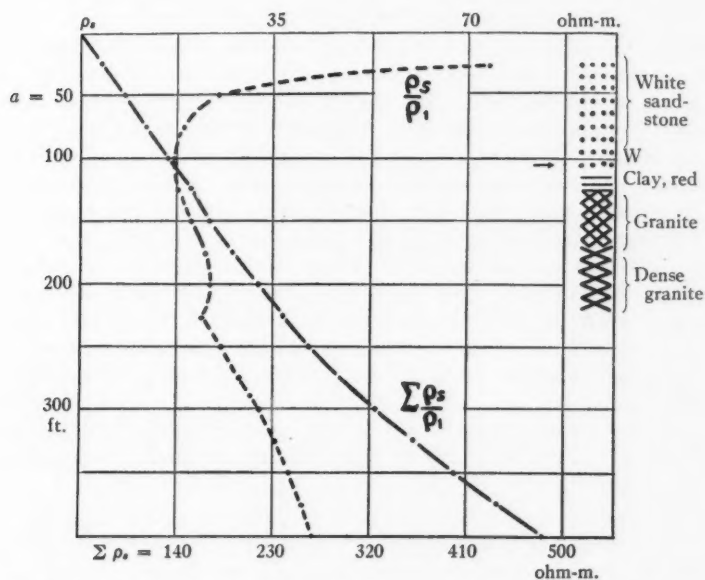

FIG. 5.   *Earth-resistivity curve and cumulative resistance for a location with relatively low resistivity in Matabeleland (Southern Rhodesia), in sandstone, clay, and granite (8).*

and moist at the bottom, may behave as two entirely distinct layers.   The conductivity of water in clay exceeds that of water in gravel so that when dry sand is underlain by wet sand, and the wet material rests on clay or on impervious rock and its moisture is stored up, exploration by the resistivity method ought to reveal three instead of two different beds.   Fig. 4, copied from a report on water-supply problems in the Nata Native Reserve in Matabeleland, near Bulawayo, Southern Rhodesia (8), and supplemented by the addition of the cumulative resistance curve, shows such a break to occur in sandstone, at a depth of between 50 and 100 ft., the average electrode spacing being increased in steps of 50 ft.   A second change in direction, equivalent to a marked increase in resistance, occurs 300 ft. below the surface.   A bore-hole put down at this site to test the occurrence of water showed sand-stone between 0 and 164 ft., broken and decomposed granite between 164 and 204 ft., and hard granite between 204 and 297 ft.   The hole yielded a water supply of 120 gal. per hr. from a depth of 170 feet, and 600 gal. per hr. at 280 ft.   The part with the least slope in the cumulative resistance curve corresponds to an extended water-bearing horizon.   Grounding would be a difficult task if only the upper 50 ft. of the soil were considered, since the resistivity exceeds 200 ohm-m., but when electrodes are spaced about 100 ft. apart, the resistance between them is determined by the ground water horizon for which the resistivity is only about 30 ohm-m.   In the whole area tested, Kalahari sandstones overlie Karroo basalt and sandstones, upon granites, and water is found when the sandstone beds are thick.

When a porous bed rests on clay, the composite resistance is likely to increase at first as the electrode separation is made larger, but it begins to fall when the spacing approaches the depth at which the water is prevented by the clay from descending to greater depth.   Provided that there is sufficient precipi-tation, structures of this type retain large quantities of water and offer most favourable conditions for grounding.   The clay acts like a dense sponge that keeps the neighbouring beds wet.   Under these conditions the resistance is relatively lower the farther apart the electrodes or grounding devices are placed.   Sand and gravel upon clay will be found where rivers discharged into old lake beds or bays.   As shown in Fig. 5 the presence of clay is revealed by good conductivity.   Drilling near the site in question, about four miles to the northwest of the spot where only sandstone and granite were encountered (Fig. 4), revealed sand between 0 and 3 ft.; rubble, 3 to 10 ft.; white sandstone, 10 to 114 ft.; red clay, 114 to 122 ft.; weathered granite, 122 to 142 ft.; granite, 142 to 169 ft.; dense granite, 169 to 222 ft.   The drill hole yielded a water supply of 100 gal. per hr. at 105 ft. depth, the water standing in the hole at 92 ft. depth.

Lignite, when wet, proves to be an even better conductor than clay (9). When moist gravel is underlain by clay, and the clay by lignite, the clay behaves as a poor conductor in comparison with wet lignite, and the earth-resistivity curve shows increasing resistance with increasing depth until the spacing is comparable with the depth at which the lignite bed begins (Fig. 6).

The graph showing cumulative resistances (from determinations in which the spacings were increased in steps of one metre) exhibits changes in slope near 3 m. depth (moist surface layer), an increase in slope between 3 and 7 m.
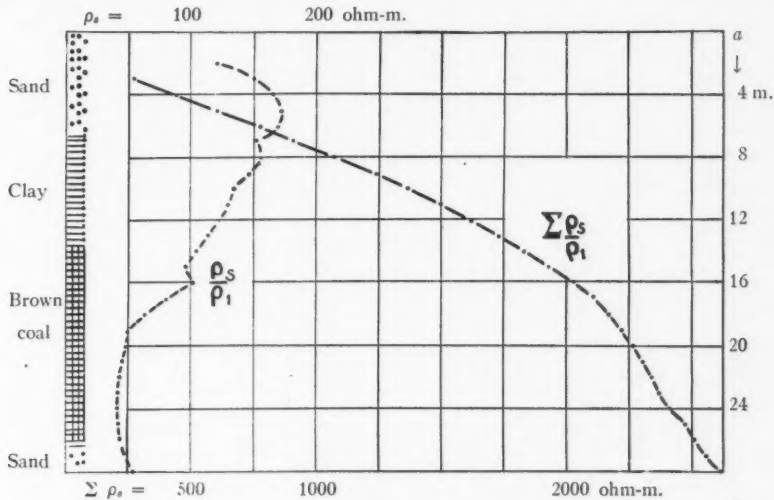

FIG. 6. *Earth-resistivity curve (shape of numeral 7) and cumulative resistance for a region where a layer of sand rests upon clay and the clay upon brown coal (9).*
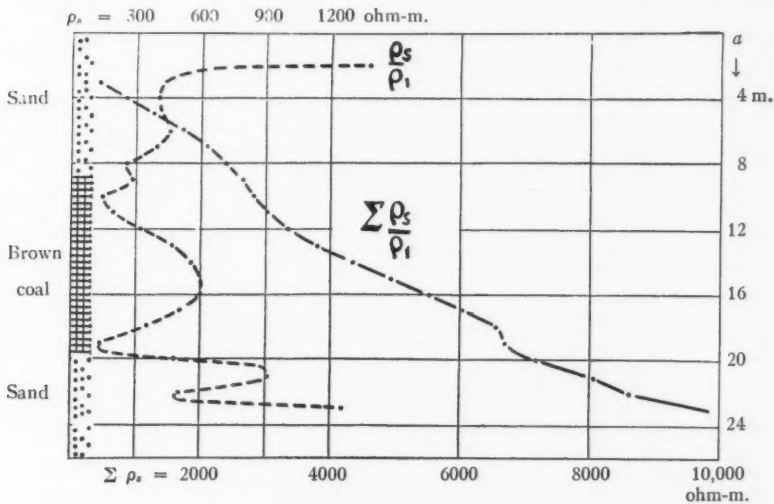

FIG. 7. *Earth-resistivity curve (roughly C-shaped) and cumulative resistances when a layer of sand is underlain by brown coal (9) and the brown coal rests on sand.*

(moist gravel), decreased slope between about 7 and 12 m., and a still smaller resistance below 12 m. Drilling reveals gravel down to 7 m. depth, clay between 7 and $13\frac{1}{2}$ m., and lignite down to at least 25 m.

In the absence of clay, the upper portion of the earth-resistivity curve reveals the effect of the low resistivity of wet lignite upon the composite resistance at short distances. (Fig. 7). For lack of an impervious bottom the water flows downward into and through the lignite. The cumulative resistance shows a steep slope for the gravel down to $7\frac{1}{2}$ or 8 m. depth, a moderate slope for the upper part of the lignite bed, an increased slope for the lower part of the lignite, and an additional break at a depth between 18 and 20 m. Drilling indicates that the lignite bed ends 19 m. below the surface (9).

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